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受压旋转梁在弹性支承双参数激励作用下的振动特性

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摘 要:研究了受压旋转细长梁在弹性支承下的参数动力学行为。针对受恒轴力作用的旋转 Rayleigh 梁,考虑其一端弹性支承和另一端简支约束,采用 Hamilton 原理得到了此旋转梁的偏微分运动方程,通过伽辽金法得到了广义模态坐标的运动方程。采用多尺度法分析了此受压旋转 Rayleigh 梁在弹性支承激励下的参数振动特性。分析了梁在不同参数下的失稳类型与失稳的形式,绘制了系统在弹性支承参数激励下的稳定图,着重探讨了弹性支承边界对梁系统稳定区造成的影响。结果显示系统的参数共振包含两种形式:主共振和组合共振。稳定图反映了转速和外加载荷在弹性支承激励下对系统的稳定区的影响。

关键词:弹性支承;旋转 Rayleigh 梁;多尺度法;参数激励

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Vibration characteristics of a compressed spinning beam under the action of two-parameter excitation of elastic support

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Abstract: The parametric dynamical behavior of a spinning slender beam under elastic support is studied. A spinning Rayleigh beam under constant axial force is considered, sustained by an elastic support at one end and a simply support at the other end. The dynamical model is obtained based on the Hamilton principle, and the motion equations for the generalized mode coordinates are derived through the Galerkin method. The parametric vibration characteristics of the spinning Rayleigh beam with elastic support are analyzed according to the multiple scale method. The instability type and form of the beam under different parameters are analyzed, and the stability diagram of the system under the excitation of elastic support parameters is drawn. The effect of the elastic support boundary on the stability zone of the beam system is emphasized. The results show that the system contain two types of parametric resonance: main resonance and combined resonance. The stability diagram reflects the influence of the speed and the external load on the stability zone of the system under the excitation of the elastic support.

Key words: elastic support; spinning Rayleigh beam; multiple scale method; parametric excitation

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钻井平台和船舶传动装置、直升机旋转部件以及卫星等航天器内部旋转系统中,存在着许多能简化为弹性支承的细长旋转梁的结构。因此,对该类结构进行充分的研究[1-4]显得尤为重要。

目前,对简支和固支边界或者轴向力作用下的旋转梁的自由和参数振动、固有频率以及临界转速已有大量研究[5-9]。MAO 等[10]建立了旋转 Euler-Bernoulli 梁的控制方程,并利用修正分解法(adomian modified decomposition method,AMDM)求得在不同边界与不同转速下的梁的固有频率。BANERJEE 等[11]基于哈密顿原理并考虑弯扭耦合作用得到不同边界下旋转复合梁的动态刚度矩阵,应用 Wittrick-Williams 算法求得梁在不同转速下的固有频率。ASGHARI 等[12]对三维微旋转 Rayleigh 梁的非线性耦合振动进行研究,通过哈密顿原理和修正的耦合应力理论,推导出微旋转 Rayleigh 梁的非线性运动方程,求解得到了梁的固有频率,并讨论了尺度效应对固有频率与临界转速的影响。SHEU 等[13]在两种坐标系下,对 Euler-Bernoulli 梁和 Rayleigh 梁在旋转条件下的动态特性进行对比分析,并分析出旋转情况下 Rayleigh 梁的动态响应更加准确。HOSSEINI 等[14]考虑几何非线性效应,建立普通边界条件下旋转梁运动方程并采用多尺度法求解,并探讨自由振动与主共振中,正反模态激发的状态。FRANK 等[15]研究了在不同边界下旋转 Rayleigh 梁线性与非线性模型的差异,进一步分析转动惯量、离心力、转速等因素对临界转速的影响。熊春等[16]讨论了旋转梁在轴向参数激励下的振动行为,着重分析轴向力,长细比等参数对系统稳定性的影响。

综上,针对旋转梁在同时存在弹性支承和外加载荷作用下的振动特性的研究少见报道。本研究主要讨论轴向受压的旋转 Rayleigh 梁在弹性支承边界作用下的参数振动稳定性问题。采用哈密顿原理和伽辽金法进行系统建模,基于多尺度法分析系统的参激振动特性,得到共振条件和类型。理论推导与数值验证相结合共同揭示了系统在不同参数域内的动力学特性。

2 建立动力学模型

考虑一轴向受压旋转细长梁,梁长为 l,材料密度为 ρ,刚度为 EI,转速为 Ω,轴向载荷为 P_t,截面面积为 A;梁 Z 与 Y 方向上的位移分别为 $W(X,t)$、$V(X,t)$;XZ 与 XY 面内的转角分别:$\theta = \partial W(X,t)/\partial X$、$\Phi = \partial V(X,t)/\partial X$。设梁左侧为简支约束,右侧为弹性支承,在 Y 与 Z 方向的弹性系数均为 K。

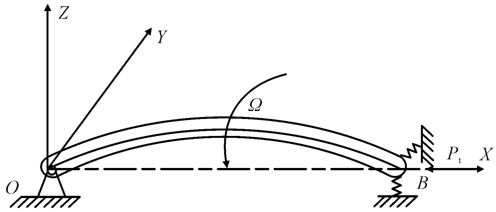


图 1 受压弹性支承旋转梁模型

Fig. 1 Model of spinning beam with elastic support under compression

通过 Hamilton 原理 $\delta\int_{t_1}^{t_2}(T-U+W_f)\,\mathrm{d}t=0$ 得到系统非线性偏微分运动方程,即

$$
\begin{cases}
EI\dfrac{\partial^4 V}{\partial X^4}-\dfrac{1}{2}EA\left[\begin{array}{l}3\left(\dfrac{\partial V}{\partial X}\right)^2\dfrac{\partial^2 V}{\partial X^2}+\dfrac{\partial^2 V}{\partial X^2}\left(\dfrac{\partial W}{\partial X}\right)^2\\ +2\dfrac{\partial V}{\partial X}\dfrac{\partial W}{\partial X}\dfrac{\partial^2 W}{\partial X^2}\end{array}\right]+\\
\rho A\dfrac{\partial^2 V}{\partial t^2}+P_t\dfrac{\partial^2 V}{\partial x^2}-\rho I\left(\begin{array}{l}\dfrac{\partial^4 V}{\partial X^2\partial t^2}+2\Omega\dfrac{\partial^3 W}{\partial X^2\partial t}\\ -\Omega^2\dfrac{\partial^2 V}{\partial X^2}\end{array}\right)=0\\
EI\dfrac{\partial^4 W}{\partial X^4}-\dfrac{1}{2}EA\left[\begin{array}{l}3\left(\dfrac{\partial W}{\partial X}\right)^2\dfrac{\partial^2 W}{\partial X^2}+\left(\dfrac{\partial V}{\partial X}\right)^2\dfrac{\partial^2 W}{\partial X^2}\\ +2\dfrac{\partial V}{\partial X}\dfrac{\partial^2 V}{\partial X^2}\dfrac{\partial W}{\partial X}\end{array}\right]+\\
\rho A\dfrac{\partial^2 W}{\partial t^2}+P_t\dfrac{\partial^2 W}{\partial X^2}-\rho I\left(\begin{array}{l}\dfrac{\partial^4 W}{\partial X^2\partial t^2}-2\Omega\dfrac{\partial^3 V}{\partial X^2\partial t}\\ -\Omega^2\dfrac{\partial^2 W}{\partial X^2}\end{array}\right)=0
\end{cases}
\tag{1}
$$

边界条件为:左侧简支 $X=0$ 时,$V=0$,$W=0$;右侧弹性支承 $X=L$ 时,$KV=F_{BY}$,$KW=F_{BZ}$,其中 F_{BY} 和 F_{BZ} 为 Y、Z 方向的弹性力。

引入无量纲参数:$x=X/l$;$\tau=\sqrt{EI/\rho A}t/l^2$;$v=V/l$;$w=W/l$;$\beta_1^*=\beta_1 l$;$r^*=\sqrt{I/A}/l$;$\beta_1=\pi/l$;$P=(l^2/EI)P_t$;$\Omega^*=\Omega/(\beta_1^2\sqrt{EI/\rho A})$;$k=Kl^3/EI$。代入式(1)得到系统的无量纲非线性方程为

$$
\begin{cases}
v''''-r^{*2}(\ddot{v}''+2\beta_1^{*2}\Omega^*\dot{w}''-\beta_1^{*4}\Omega^{*2}v'')+\ddot{v}+Pv''\\
-\dfrac{1}{2r^{*2}}[3(v')^2v''+v''(w')^2+2v'w'w'']=0\\
w''''-r^{*2}(\ddot{w}''-2\beta_1^{*2}\Omega^*\dot{v}''-\beta_1^{*4}\Omega^{*2}w'')+\ddot{w}+Pw''\\
-\dfrac{1}{2r^{*2}}[3(w')^2w''+w''(v')^2+2w'v'v'']=0
\end{cases}
\tag{2}
$$

式中:“′”表示 $\partial(\ )/\partial x$;“·”表示 $\partial(\ )/\partial\tau$ 。

设梁的无量纲位移函数为

$$\begin{cases} v = v_r + v_d = v_0 x + q_1(\tau)\sin(n\pi x) \\ w = w_r + w_d = w_0 x + q_2(\tau)\sin(n\pi x) \end{cases} \tag{3}$$

式中,v_d 和 w_d 为梁的变形位移,$v_r = v_0 x$ 和 $w_r = w_0 x$ 为弹性支承导致的梁的刚体位移。设 $\lambda = n\pi$ 为梁的模态参数,忽略式(2)中的非线性项,采用伽辽金积分,得到关于模态坐标 q_1 和 q_2 的常微分运动方程组,即

$$\begin{cases} \ddot{q}_1 + \beta q_1 - f q_1 + \xi_1 q_1 + \gamma \dot{q}_2 + n_1 q_2 = 0 \\ \ddot{q}_2 + \beta q_2 - f q_2 + \xi_2 q_2 - \gamma \dot{q}_1 + n_2 q_1 = 0 \end{cases} \tag{4}$$

式中: $\gamma = 2g\beta_1^{*2} r^{*2} \Omega^{*2}$;$\beta = g(\lambda^2 - \beta_1^{*4} r^{*2} \Omega^{*2})$;$f = gP$;$g = \lambda^2/(r^{*2}\lambda^2 + 1)$;$\alpha = g/r^{*2}$ 。

假设弹性支承位移,$v_0 = h\cos(\omega_0\tau)$,$w_0 = h\cos(\omega_0\tau)$,其中 h 表示无量纲弹簧振幅,则

$\xi_1 = \xi_2 = 2gh^2\cos^2(\omega_0\tau)/r^{*2} = 2N\cos^2(\omega_0\tau) = N + N\cos(2\omega_0\tau)$,

$n_1 = n_2 = h^2\alpha\cos^2(\omega_0\tau) = \xi/2 = N/2 + N\cos 2(\omega_0\tau)/2$

式中: ω_0 为弹性激励频率; $N = h^2\alpha$,用于表征弹性激励幅值。

3　方程解耦与稳定性分析

3.1　多尺度法与共振类型判断

为考虑弹性参数对系统固有频率的影响,仅在时变参数激励项中引入小参数 ε,使 $N \to \varepsilon N$。此时,关于 q_1、q_2 均有激励项产生,为双参数激励。将式(4)整理后得到

$$\begin{cases} \ddot{q}_1 + (\beta - f + N) q_1 + \gamma \dot{q}_2 + N q_2/2 + \\ \varepsilon N\cos(2\omega_0\tau) q_1 + \varepsilon N\cos(2\omega_0\tau) q_2/2 = 0 \\ \ddot{q}_2 + (\beta - f + N) q_2 - \gamma \dot{q}_1 + N q_1/2 + \\ \varepsilon N\cos(2\omega_0\tau) q_1/2 + \varepsilon N\cos(2\omega_0\tau) q_2 = 0 \end{cases} \tag{5}$$

设式(5)的解可表示为级数形式

$$q_m = q_{m0}(T_0, T_1) + \varepsilon q_{m1}(T_0, T_1) \quad (m = 1,2) \tag{6}$$

式中,变量 $T_i = \varepsilon^i \tau\ (i = 0,1)$,设微分算子 $D_i = \partial(\ )/\partial T_i$ 。

将式(6)代入方程(5),根据 ε 同幂次项系数相加为0,ε^0 项系数合并得到

$$\begin{cases} D_0^2 q_{10} + (\beta - f + N) q_{10} + \gamma D_0 q_{20} + N q_{20}/2 = 0 \\ D_0^2 q_{20} + (\beta - f + N) q_{20} - \gamma D_0 q_{10} + N q_{10}/2 = 0 \end{cases} \tag{7}$$

由式(7)得到特征方程

$$\omega^4 - [\gamma^2 + 2(\beta - f + N)]\omega^2 + (\beta - f + N)^2 - N^2/4 = 0 \tag{8}$$

弹性支承受压旋转梁系统的固有频率为

$$\omega_{1,2} = \sqrt{\frac{[\gamma^2 + 2(\beta - f + N)] \pm \sqrt{[\gamma^2 + 2(\beta - f + N)]^2 - 4[(\beta - f + N)^2 - N^2/4]}}{2}} \tag{9}$$

式中,$0 < \omega_2 < \omega_1$ 。

从固有频率的表达式易知旋转梁系统的固有频率与转速、外加载荷和弹性支承约束均有关。

ε^1 项系数合并得到

$$\begin{cases} D_0^2 q_{11} + (\beta - f + N) q_{11} + \gamma D_0 q_{21} + N q_{21}/2 = \\ -2D_0 D_1 q_{10} - \gamma D_1 q_{20} - N\cos(2\omega_0 T_0) q_{10} - \\ \cos(2\omega_0 T_0) N q_{20}/2 \\ D_0^2 q_{21} + (\beta - f + N) q_{21} - \gamma D_0 q_{11} + N q_{11}/2 = \\ -2D_0 D_1 q_{20} + \gamma D_1 q_{10} - N\cos(2\omega_0 T_0) q_{20} - \\ \cos(2\omega_0 T_0) N q_{10}/2 \end{cases} \tag{10}$$

设式(7)的解的形式如下

$$\begin{cases} q_{10} = A_1(T_1) e^{i\omega_1 T_0} + A_2(T_1) e^{i\omega_2 T_0} + cc \\ q_{20} = i\Lambda_1 A_1(T_1) e^{i\omega_1 T_0} + i\Lambda_2 A_2(T_1) e^{i\omega_2 T_0} + cc \end{cases} \tag{11}$$

式中:A_1、A_2 待定的复值函数;Λ_1、Λ_2 为常数;cc 表示等号右侧给出项的共轭复数。

将式(11)代入式(10),并采用欧拉变换 $\cos(\omega_0 T_0) = \dfrac{e^{i\omega_0 T_0} + e^{-i\omega_0 T_0}}{2}$,得

$$\begin{cases} D_0^2 q_{11} + \gamma D_0 q_{21} + (\beta - f + N) q_{11} + N q_{21}/2 = \\ -i(2\omega_1 + \gamma\Lambda_1) D_1 A_1 e^{i\omega_1 T_0} - (K_1 + K_2) N/2 - \\ (K_1\Lambda_1 + K_2\Lambda_2) iN/4 - i(2\omega_2 + \gamma\Lambda_2) D_1 A_2 e^{i\omega_2 T_0} \\ D_0^2 q_{21} - \gamma D_0 q_{11} + (\beta - f + N) q_{21} + N q_{11}/2 = \\ (2\Lambda_1\omega_1 + \gamma) D_1 A_1 e^{i\omega_1 T_0} - (K_1\Lambda_1 + K_2\Lambda_2) iN/2 - \\ (K_1 + K_2) N/4 + (2\Lambda_2\omega_2 + \gamma) D_1 A_2 e^{i\omega_2 T_0} \end{cases} \tag{12}$$

式中

$$K_m = A_m[e^{i(2\omega_0 + \omega_m)T_0} + e^{i(-2\omega_0 + \omega_m)T_0}],$$

$\Lambda_m = [(\beta - f + N) - \omega_m^2]/\gamma\omega_m \quad (m = 1,2)$

由式(12)右侧的激励频率与系统频率的关系可得到系统存在两种共振类型：主共振($\omega_0 \approx \omega_1$、$\omega_0 \approx \omega_2$)、组合共振$\omega_0 \approx (\omega_1 - \omega_2)/2$、$\omega_0 \approx (\omega_1 + \omega_2)/2$)。

设式(12)解的形式为

$$\begin{cases} q_{11} = R_1(T_1)\mathrm{e}^{i\omega_1 T_0} + S_1(T_1)\mathrm{e}^{i\omega_2 T_0} + cc \\ q_{21} = R_2(T_1)\mathrm{e}^{i\omega_1 T_0} + S_2(T_1)\mathrm{e}^{i\omega_2 T_0} + cc \end{cases} \tag{13}$$

式中，R_1、R_2、S_1、S_2为复函数。

将式(13)代入式(12)左边，得

$$\begin{cases} D_0^2 q_{11} + \gamma D_0 q_{21} + (\beta - f + N)q_{11} + Nq_{21}/2 = \\ [(\beta - f + N - \omega_1^2)R_1 + (i\gamma\omega_1 + N/2)R_2]\mathrm{e}^{i\omega_1 T_0} + \\ [(\beta - f + N - \omega_2^2)S_1 + (i\gamma\omega_2 + N/2)S_2]\mathrm{e}^{i\omega_2 T_0} \\ D_0^2 q_{21} - \gamma D_0 q_{11} + (\beta - f + N)q_{21} + Nq_{11}/2 = \\ [(\beta - f + N - \omega_1^2)R_2 - (i\gamma\omega_1 - N/2)R_1]\mathrm{e}^{i\omega_1 T_0} + \\ [(\beta - f + N - \omega_2^2)S_2 - (i\gamma\omega_2 - N/2)S_1]\mathrm{e}^{i\omega_2 T_0} \end{cases} \tag{14}$$

3.2 参数振动下的主共振与组合共振

3.2.1 主共振

3.2.1.1 $\omega_0 \approx \omega_1$

设谐调参数σ_1[17-18]，令$\omega_0 = \omega_1 + \varepsilon\sigma_1$成立，联立式(12)和式(14)，根据各对应指数函数系数相等，得

$$\begin{cases} (\beta - f + N - \omega_1^2)R_1 + (i\gamma\omega_1 + N/2)R_2 = -i(2\omega_1 + \\ \gamma\Lambda_1)D_1A_1 - \bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}N/2 - \Lambda_1\bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}iN/4 \\ (N/2 - i\gamma\omega_1)R_1 + (\beta - f + N - \omega_1^2)R_2 = (2\Lambda_1\omega_1 + \\ \gamma)D_1A_1 - \Lambda_1\bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}iN/2 - \bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}N/4 \end{cases} \tag{15}$$

$$\begin{cases} (\beta - f + N - \omega_2^2)S_1 + (N/2 + i\gamma\omega_2)S_2 = -i(2\omega_2 + \\ \gamma\Lambda_2)D_1A_2 \\ (N/2 - i\gamma\omega_2)S_1 + (\beta - f + N - \omega_2^2)S_2 = (2\Lambda_2\omega_2 + \\ \gamma)D_1A_2 \end{cases} \tag{16}$$

由线性方程解的存在性，得

$$\begin{bmatrix} \beta - f + N - \omega_1^2 & -i(2\omega_1 + \gamma\Lambda_1)D_1A_1 - \bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}N/2 - \Lambda_1\bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}iN/4 \\ N/2 - i\gamma\omega_1 & (2\Lambda_1\omega_1 + \gamma)D_1A_1 - \Lambda_1\bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}iN/2 - \bar{A}_1\mathrm{e}^{i2\sigma_1 T_1}N/4 \end{bmatrix} = 0$$

$$\begin{bmatrix} \beta - f + N_0 - \omega_2^2 & -i(2\omega_2 + \gamma\Lambda_2)D_1A_2 \\ N/2 - i\gamma\omega_2 & (2\Lambda_2\omega_2 + \gamma)D_1A_2 \end{bmatrix} = 0$$

可得

$$D_1A_1 - i\varphi_1\bar{A}_1\mathrm{e}^{i2\sigma_1 T_1} - \varphi_2\bar{A}_1\mathrm{e}^{i2\sigma_1 T_1} = 0 \tag{17}$$

$$D_1A_2 = 0 \tag{18}$$

φ_1和φ_2式子过于冗长，见附录(A1)和(A2)。

令$A_1 = (B_{r_1} + iB_{i_1})\exp(i\sigma_1 T_1)$，代入式(17)中得

$$\begin{cases} D_1B_{i_1} + (\sigma_1 + \varphi_1)B_{r_1} - \varphi_2 B_{i_1} = 0 \\ D_1B_{r_1} - (\sigma_1 + \varphi_1)B_{i_1} - \varphi_2 B_{r_1} = 0 \end{cases} \tag{19}$$

采用文献[17-18]的求解方法，令式(19)的解为$(B_{r_1}, B_{i_1}) = (b_{r_1}, b_{i_1})\exp(\mu_1 T_1)$的形式，则有$(\mu - \varphi_2)^2 + (\sigma_1 - \varphi_1)^2 = 0$。即受压弹性支承旋转Rayleigh梁双参数激励下的临界失稳条件是

$$\omega_0 = \omega_1 \pm \varepsilon\varphi_1 \tag{20}$$

3.2.1.2 $\omega_0 \approx \omega_2$

与前文同理可得此时系统双参数激励下的临界失稳条件为

$$\omega_0 = \omega_2 \pm \varepsilon\varphi_2 \tag{21}$$

φ_3和φ_4的表达式见附录(A3)和(A4)。

3.2.2 组合共振

3.2.2.1 差型共振：$\omega_0 \approx (\omega_1 - \omega_2)/2$

同理设参数σ_2，令$\omega_0 = (\omega_1 - \omega_2)/2 + \varepsilon\sigma_2$成立，与主共振同理，根据各对应指数函数系数相等，可得

$$\begin{cases} D_1A_1 - \varphi_2 A_2\mathrm{e}^{i2\sigma_2 T_1} - i\varphi_1 A_2\mathrm{e}^{i2\sigma_2 T_1} = 0 \\ D_1A_2 - \varphi_3 A_1\mathrm{e}^{-i2\sigma_2 T_1} - i\varphi_4 A_1\mathrm{e}^{-i2\sigma_2 T_1} = 0 \end{cases} \tag{22}$$

令式(20)的解为，$A_1 = a_1\exp(2\mu_2 T_1)$，$A_2 = a_2\exp[2(\mu_2 - i\sigma_2)T_1]$，将解代入可得，$2\mu_2^2 - 2i\sigma_2\mu_2 - (\varphi_2\varphi_3 - \varphi_1\varphi_4 + i\varphi_1\varphi_3 + i\varphi_2\varphi_4) = 0$，即受压弹性支承旋转Rayleigh梁双参数激励下的临界失稳条件是

$$\omega_0 = \frac{1}{2}(\omega_1 - \omega_2) \pm \varepsilon\frac{(\varphi_1\varphi_3 + \varphi_2\varphi_4)}{\sqrt{2(\varphi_2\varphi_3 - \varphi_1\varphi_4)}} \tag{23}$$

3.2.2.2 和型共振：$\omega_0 \approx (\omega_1 + \omega_2)/2$

按照差型共振的处理方法可得

$$\omega_0=\frac{1}{2}(\omega_1+\omega_2)\pm\varepsilon\frac{(\varphi_1\varphi_3+\varphi_2\varphi_4)}{\sqrt{2(\varphi_2\varphi_3-\varphi_1\varphi_4)}}\tag{24}$$

4　算例及结果讨论

本节给定细长梁长为 1.5 m，横截面为空心圆截面，内径为 0.02 m，外径为 0.1 m，密度为 7 850 kg/m^3，杨氏模量为 200 GPa，边界条件为一端简支一端弹性支承，以弹性参数激励 N 为纵坐标，以激励频率 w_0 为横坐标，绘制稳定图并讨论其参数振动特性。

图 2 展示了在无量纲轴向力 $P=0.8$ 时，弹性参数激励作用下不同无量纲转速下的稳定图。结果表明：在弹性参数激励下，（无量纲）转速对失稳定区的位置和大小都有影响；随着转速的增大，主共振不稳定区域向左移动，主共振不稳定区面积逐渐扩大，差型共振不稳定区在加速右移，大小逐渐变大，而和型共振不稳定区在加速左移，逐渐增大。

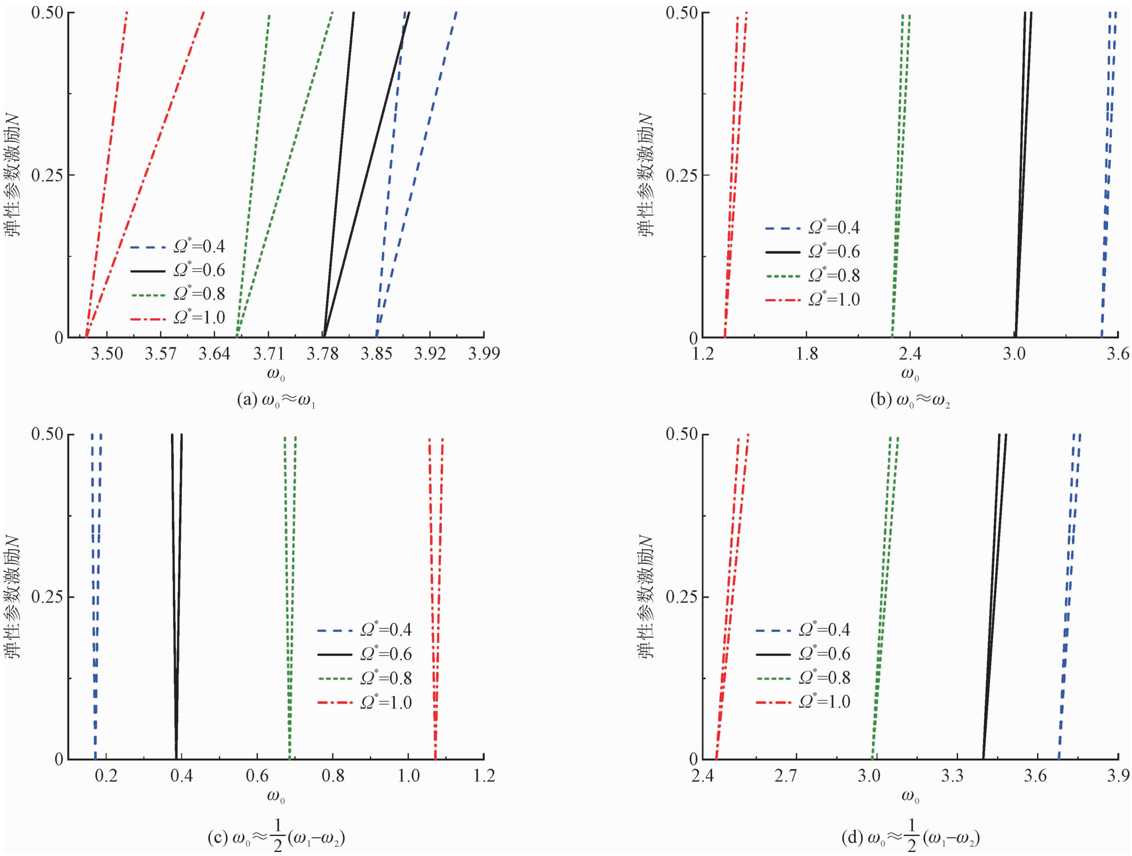


图 2　不同无量纲转速下的参数激励稳定图

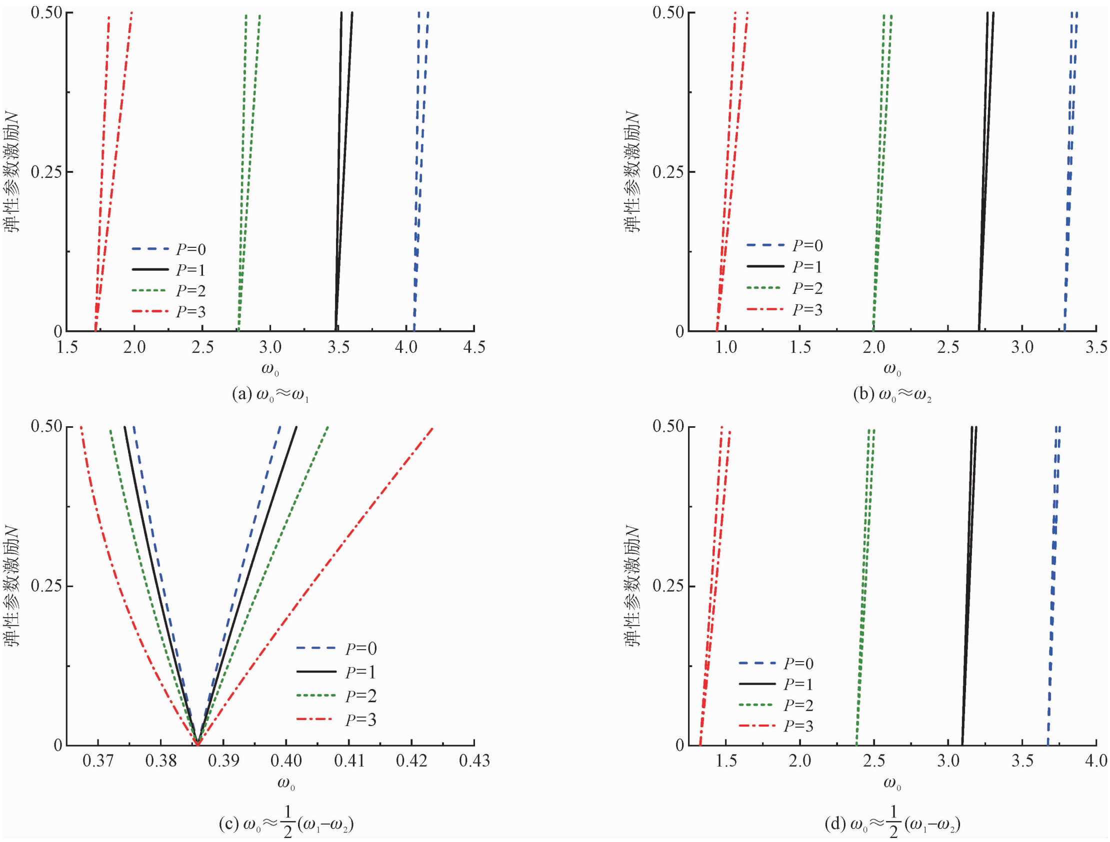
Fig. 2　Parameter excitation stability diagram under different dimensionless speeds

图 3 展示了在无量纲转速 $\Omega^*=0.6$ 时，弹性参数激励作用下的不同（无量纲）轴向载荷下系统的稳定图。图中显示：主共振失稳区的位置随着轴向载荷的增加逐渐向左移动；且失稳区域大小逐渐增大，而差型共振失稳区也逐渐增大但其失稳区下端点的位置不变，和型共振失稳区有加速左移的趋势，区域由小变大。

根据式（8）由韦达定理，$(\omega_1-\omega_2)^2=\omega_1^2+\omega_2^2-2\sqrt{\omega_1^2\times\omega_2^2}=\gamma^2$。改变轴向载荷不影响 γ 的大小，改变转速会使 γ 变化，所以无量纲轴向载荷下差型共振失稳区下端点的位置不变，如图 3（c）所示系统的稳定图。

采用数值解法，对耦合常微分方程组（5）进行数值模拟，检验弹性参数激励作用下的无量纲转速下的稳定图，对两种共振类型下的共振情况分别检验。

图 4 显示了受压弹性支承旋转梁系统的稳定图，图中 U_i（空白区域）、S_i（$i=1,2,3$）（阴影区域）分别表示失稳区和稳定区，负数区不需检验，检测 U_1、U_2、S_2、S_3 图中部分区域，在图中区域选取点，代入仿真程序中计算。图 5（a、b、c、d）展现了检验成果，数值方法表明近似解是正确的。



(a) $\omega_0 \approx \omega_1$

(b) $\omega_0 \approx \omega_2$

(c) $\omega_0 \approx \frac{1}{2}(\omega_1 - \omega_2)$

(d) $\omega_0 \approx \frac{1}{2}(\omega_1 - \omega_2)$

图 3 不同无量纲轴向载荷下的参数激励稳定图

Fig. 3 Parametric excitation stability diagram under different dimensionless axial loads

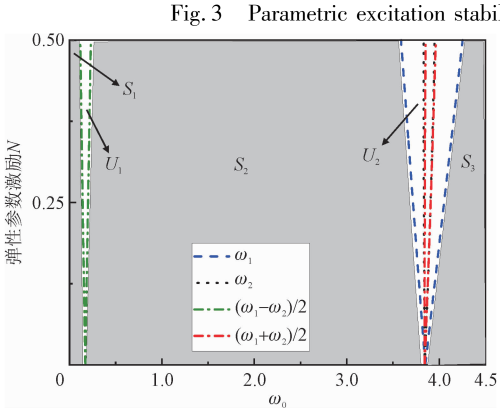
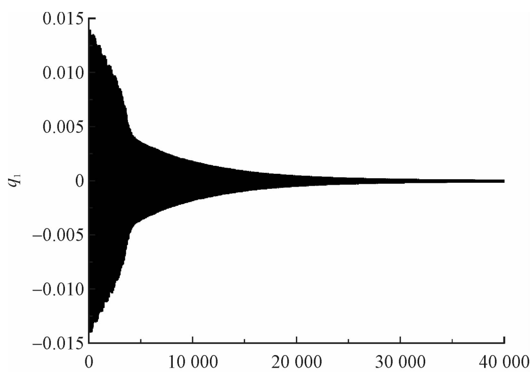
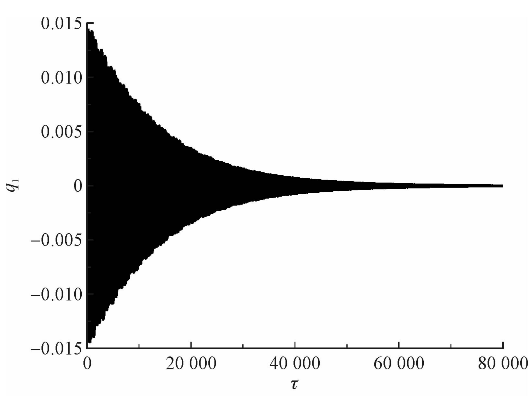


图 4 主共振与组合共振的稳定图

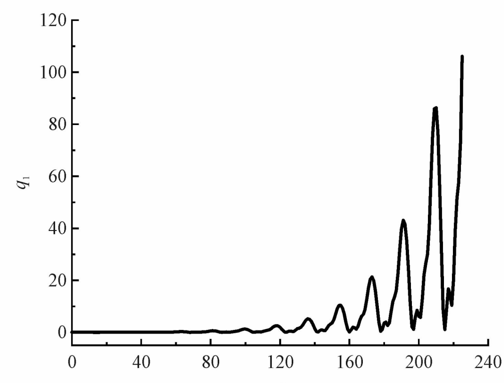
Fig. 4 Stability diagram of main resonance and combined resonance



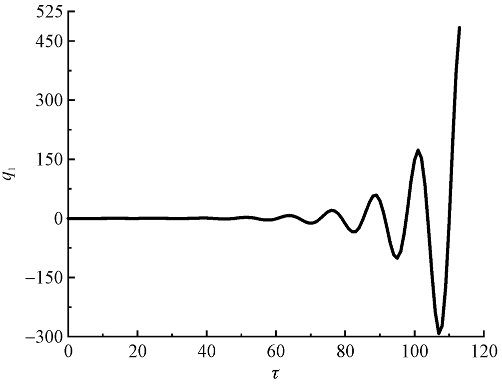
(b) S_3



(a) S_2



(c) U_1



(d) U_2

图 5　失稳区与稳定区的时程曲线图

Fig. 5　Time history curve diagram of instability zone and stable zone

5　结　论

理论方法(Hamilton 原理与多尺度法)和数值方法相结合,讨论了受压弹性支承下的旋转梁的双参数激励振动特性,并得到如下结论。

1)转速、轴向压力和弹性支承边界约束都彼此对旋转梁系统的固有频率产生影响。

2) 受压弹性支承的旋转 Rayleigh 梁的参数振动存在主共振、组合共振两种共振形式。

3) 随着(无量纲)转速的增大,促使系统向着失稳方向发展:主共振系统的不稳定区域逐渐向左移动,不稳定区大小变大。差型共振不稳定区在加速右移,大小增大,而和型共振不稳定区在加速左移,大小增大。

4) 随着(无量纲)轴向载荷的增大,也使系统向着失稳方向发展,主共振中系统的不稳定区域向左移动,大小变大。差型共振失稳区增大且其失稳区下端点的位置不变,和型共振失稳区有加速左移的趋势,区域由小变大。

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附录

多尺度计算中临界条件里的 φ_1、φ_2、φ_3、φ_4 的形式为

$$\varphi_1 = \frac{1}{C_1}\left(\frac{N}{4}G_1 - \frac{N}{2}\Lambda_1 E_1\right)H_1 + \frac{N^2}{8C_1}(E_1 - 2G_1) - \frac{N}{4C_1}\gamma\omega_1(2E_1 + G_1) \tag{A1}$$

$$\varphi_2 = -\frac{1}{C_1}\left(\frac{N}{4}E_1 + \frac{N}{2}\Lambda_1 G_1\right)H_1 + \frac{N^2}{8C_1}(2E_1 + G_1) + \frac{N}{4C_1}\gamma\omega_1(E_1 - 2G_1) \tag{A2}$$

$$\varphi_3 = -\frac{1}{C_2}\left(\frac{N}{4}E_2 + \frac{N}{2}\Lambda_2 G_2\right)H_2 + \frac{N^2}{8C_2}(2E_2 + G_2) + \frac{N}{4C_2}\gamma\omega_2(E_2 - 2G_2) \tag{A3}$$

$$\varphi_4 = \frac{1}{C_2}\left(\frac{N}{4}G_2 - \frac{N}{2}\Lambda_2 E_2\right)H_2 + \frac{N^2}{8C_2}(E_2 - 2G_2) - \frac{N}{4C_2}\gamma\omega_2(2E_2 + G_2) \tag{A4}$$

E_m、G_m、C_m、H_m的表达式为

$$\begin{cases} E_m = (\beta - f + N - \omega_m^2)(2\Lambda_m\omega_m + \gamma) + \gamma\omega_m(2\omega_m + \gamma\Lambda_m) \\ G_m = N/2(2\omega_m + \gamma\Lambda_m) \\ C_m = E_m^2 + G_m^2 \\ H_m = (\beta - f + N - \omega_m^2) \end{cases} \quad (m = 1,2)$$